Example

A train of mass 112 tons and the driving force of its engine is 5600 kg . wt . If the resistance to the motion of the train is directly proportional to the square of its velocity and this resistance is 32 kg . wt for each ton of the mass when its velocity was 60 km / h. Calculate the maximum velocity of the train .

$$R \propto V^2$$

$$\therefore \quad \frac{R_1}{R_2} = \frac{V_1^2}{V_2^2}$$

$$\therefore \quad \frac{32 \times 112}{5600} = \frac{60^2}{V_2^2}$$

$$\therefore \quad V_2 = 75 \ km / h$$

$$R_1 = 32 \times 112 \ kg.wt$$

$$V_1 = 60 \ km / h$$

$$R_2 = F = 5600$$

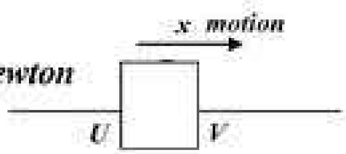
Example Newton's second law

A flying body of mass 800 kg flies in space with uniform motion with speed of 900 km/h suddenly it enters a cloud of dust which acts on it with a force of friction (resistance) whose magnitude is $\frac{1}{2}$ kg . wt for each kilogram of its mass. Find its velocity after coming out of the cloud , if it stayed inside the cloud for 20 seconds.

Solution

$$R = \frac{1}{2} \times 800 = 400 \ kg.wt$$

$$= 400 \times 9.8 = 3920 \ newton$$



$$\ldots \quad -\overline{R} = m\overline{A}$$

$$3920 \ x = 800\overline{A} \Rightarrow \overline{A} = -4.9x \quad m / sec^2$$

The body inters the cloud with velocity

$$U = 900 \ km / h = 900 \times \frac{5}{18} = 250 \ m / sec$$

$$\overline{A} = -4.9 \ m / sec^2 \quad , \quad t = 20 \ second$$

$V = U + At \Rightarrow V = 250 - 4.9 \times 20 = 152$ $m / sec = 547.2 \ km / h$

Example *A car of mass 2.5 tons started motion from rest along a st. road with uniform acceleration under the action of its motor driving force whose magnitude is 325 kg . wt . and the road resistance whose magnitude is 50 kg . wt per each ton of its mass. Find magnitude of the car velocity after 25 seconds of the start of motion. If the driving engine is stopped after this instant , and the magnitude of the resistance didn't change. Find the time taken after this instant until the car comes to rest.*

Solution

$$F = 325 \ kg . wt = 325 \times 9.8 = 3185 \ N$$

$$R = 2.5 \times 50 \ kg . wt = 2.5 \times 50 \times 9.8 = 1225 \ N$$

$$\ldots \quad F - R = mA$$

$$3185 - 1225 = 2500 \ A$$

$$\Rightarrow \quad A = \frac{98}{125} m / sec^2$$

after 25 seconds $\quad V = U + At$

$$V = 0 + \frac{98}{125} \times 25 = 19.6 \ m / sec$$

When the engine is stopped

$-R = mA^{\backprime}$

$-1225 = 2500\,A^{\backprime} \Rightarrow A^{\backprime} = -0.49\,m/sec^2$

$U = 19.6 \quad , \quad V = 0 \quad , \quad A^{\backprime} = -0.49$

$V = U + At$

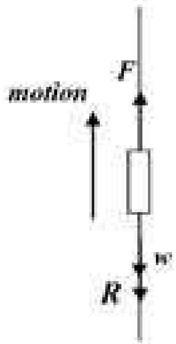
$0 = 19.6 - 0.49 \times t \Rightarrow t = 40\,sec$

Example A helicopter of mass 2 tons is ascending vertically with uniform acceleration $0.49\ m/sec^2$ against a constant resistance of magnitude 450 kg . wt per each ton of its mass. Calculate the magnitude of the lifting force of the plane motor in kg . wt.

Solution

Weight $w = mg = 2000 \times 9.8\ N$

Resistance $R = 450 \times 2 \times 9.8\ N$



$F - w - R = ma$

$F - 19600 - 8820 = 2000 \times 0.49$

$\therefore\ F = 29400$ Newton

$= 300$ kg . wt

Example A force $\overline{F}$ acts upon a body so its displacement after a time (t) is given by $\overline{S} = (\frac{4}{7}t^2 + 2t)\,\hat{c}$ where $\hat{c}$ is a unit vector in the direction of $\overline{F}$. If the momentum of this body after 3 seconds is $19\,\hat{c}$. Find the mass of this body and the magnitude of $\overline{F}$

Solution

$\overline{S} = (\frac{4}{7}t^2 + 2t)\,\hat{c}$

$\therefore\ \overline{V} = (\frac{8}{7}t + 2)\,\hat{c}$

$\overline{A} = (\frac{8}{7})\,\hat{c}$

$\because$ momentum is $19\,\hat{c}$ at $t = 3$

$19\,\hat{c} = m\overline{V}$

$\therefore\ 19\,\hat{c} = m(\frac{24}{7} + 2)\,\hat{c}$

$m = 3.5$ unit

$\overline{F} = m\overline{A} = 3.5 \times \frac{8}{7}\hat{c} = 4\,\hat{c}$ unit

Newton's third law

Example Find the reaction of a lift on a person inside it , whose mass is 70 kg in newtons in the following cases :

1) If the lift moving with a uniform velocity

2) If the lift moving with a uniform acceleration of magnitude $1.2\ m/sec^2$ vertically upwards .

3) If the lift moving with a uniform acceleration of magnitude $1.8\ m/sec^2$ vertically downwards .

Solution

<u>Case 1 :</u> $\because$ The lift moving with uniform velocity

$\therefore\ N = mg \qquad \therefore\ N = 70 \times 9.8 = 686\ N$

<u>Case 2 :</u> $\because$ the lift moving upwards with acceleration

$\therefore \; N = m(g + a)$

$\therefore \; N = 70(9.8 + 1.2) = 770$ newton

Case 3 : the lift moving downwards with acceleration

$\therefore \; N = m(g - a) = 70(9.8 - 1.8) = 560 \; N$

Example A lift ascends from rest with a uniform acceleration for 4 seconds , then with uniform velocity for 3 seconds then with uniform deceleration for 2 seconds so it comes to rest when it reaches a point 9.6 m height above the starting point. Find the magnitude of the acceleration in each of the 1^{st} and last stages, and if there is a man of mass 73.5 kg inside the lift. Find his pressure on the floor of the lift during each of the three stages.

Solution

1^{st} stage : $U = 0$, $V = ??$,

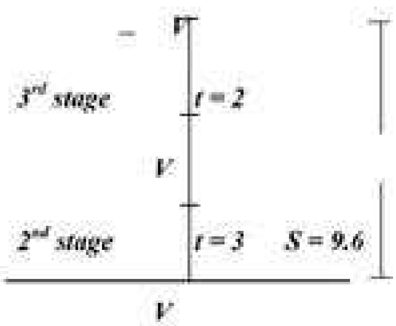
$t = 4$, acceleration $= a$

$\therefore \; V = U + a\,t$

$V = 0 + 4a \Rightarrow V = 4a$

$S_1 = Ut + \frac{1}{2}at^2$

$= 0 + \frac{1}{2}a(16) \qquad \therefore S_1 = 8a$ ____ (1)



2^{nd} stage : uniform motion with velocity $4a$

$S_2 = Vt + 4a \times 3 = 12a$ ———— (2)

3^{rd} stage : $U = 4a$, $V = 0$, $t = 2$

acceleration $= a`$ (retardation)

$\therefore \; V = U - a`t$

$0 = 4a - 2a` \qquad a` = 2a$

$S_3 = Ut - \frac{1}{2}a`t^2$

$= 4a(2) - \frac{1}{2}(2a)(4) \Rightarrow \therefore S_3 = 4a$ ——— (3)

$\because \; S_1 + S_2 + S_3 = 9.6$

$\therefore \; 8a + 12a + 4a = 9.6$

$24a = 9.6 \Rightarrow a = 0.4 \; m / sec^2$

$\therefore \; a` = -0.8 \; m / sec^2$

$\therefore$ the pressure of the man at 1^{st} stage

$N = m(g + a)$

$= 73.5(9.8 + 0.4) = 749.7 \; N$

$= 76.5$ kg.wt

The pressure at the 2^{nd} stage

$N = mg = 720.3 \; N = 73.5$ kg . wt

The pressure at 3^{rd} stage

$N = m(g + a)$

$= 73.5(9.8 - 0.8)$

$= 661.5 \; N = 67.5$ kg . wt

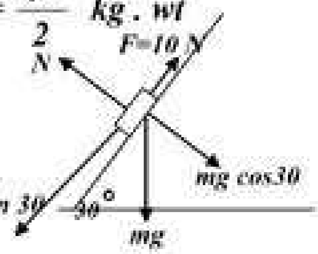
Motion of a body on a smooth inclined plane.

Example ex A body of mass 1 kg is placed on a smooth inclined plane of inclination 30° to the horizontal a force of magnitude 10 Newtons acts on the body along a line of greatest slope upwards. Find the magnitude of the force of reaction of the plane on it and find also its acceleration .

Solution

$N = mg \cos 30 \qquad = 1 \times 9.8 \times \frac{\sqrt{3}}{2}$

$= 4.9\sqrt{3}$ Newto $= \frac{\sqrt{3}}{2}$ kg . wt



$F - mg \sin 30 = ma$

$10 - = 1 \times 9.8 \times \frac{1}{2} = 1 \times a$

$a = 5.1 \text{ m/sec}^2$ / a is +ve

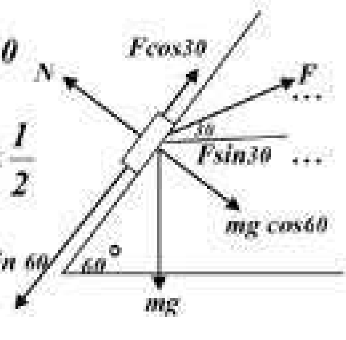
… the motion upwards

Example A body of mass 2 kg is moving along a line of greatest slope of a smooth plane inclined at an angle of measure 60° to the horizontal under the action of a force of magnitude 1 kg .wt directed towards the plane and making an angle of measure 30° with the horizontal upwards. Find the magnitude of the reaction force on the body and find also the acceleration.

Solution

$N = F \sin 30 + mg \cos 60$

$= 1 \times 9.8 \times \frac{1}{2} + 2 \times 9.8 \times \frac{1}{2}$



$= 14.7$ newton

$= 1.5$ kg . wt

Also $\quad F \cos 30 - mg \sin 60 = m a$

$1 \times 9.8 \times \frac{\sqrt{3}}{2} - 2 \times 9.8 \times \frac{\sqrt{3}}{2} = 2a$

… $a = -2.45\sqrt{3} \text{ m/sec}^2$

… the motion downwards

Example A body of mass 500 gm is placed on a smooth plane inclined to the horizontal at an angle of measure θ where $\sin\theta = \frac{3}{5}$. Force of 500 gm .wt acting parallel to the plane acts on the body. Find the acceleration of motion if the force equal zero after 2 seconds , Find the distance which the body ascends until it stops instantaneously.

Solution

$F - mg \sin\theta = ma$

$500 \times 980 - 500 \times 980 \times \frac{3}{5} = 500\, a$

… $a = 392 \text{ cm/sec}^2$

After 2 seconds :

$U = 0$, $V = ??$, $t = 2$, $a = 392$

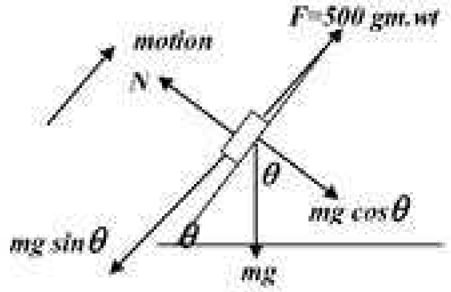
… $V = U + at$

$= 0 + 392 \times 2$ … $V = 784$ cm/sec .

When $F = 0$

$- mg \sin\theta = ma$

$a = -980 \times \frac{3}{5} = -588 \text{ cm/sec}^2$



… *The body moves with starting velocity 784 cm/sec. and acceleration - 588 cm/sec^2.*

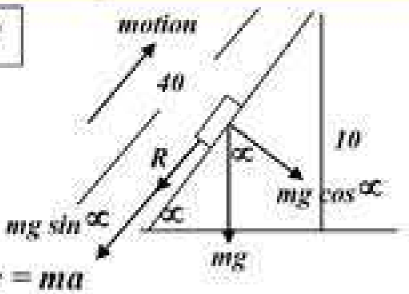
$U = 784 \quad , \quad V = 0 \quad , \quad a^{\backslash} = -588$

$\therefore \quad V^2 = U^2 + 2aS$

$$0 = (784)^2 - 2 \times 588 \times S \quad \Rightarrow \quad S = 522\frac{2}{3} cm$$

Example *A rough inclined plane its length is 40 m and its height is 10 m. If a body is projected up the plane from the bottom , Find the least velocity by which it must be projected to reach the top of the plane given that the force of the friction of the plane is $\frac{1}{4}$ of the weight of the body.*

Solution



$R = \frac{1}{4}\ mg$

$- mg \sin\propto - \frac{1}{4}\ mg = ma$

$\therefore \quad a = -9.8 \times \frac{10}{40} - \frac{1}{4} \times 9.8$

$\therefore\ a = -4.9\ m/sec^2.$

$V^2 = U^2 + 2aS$

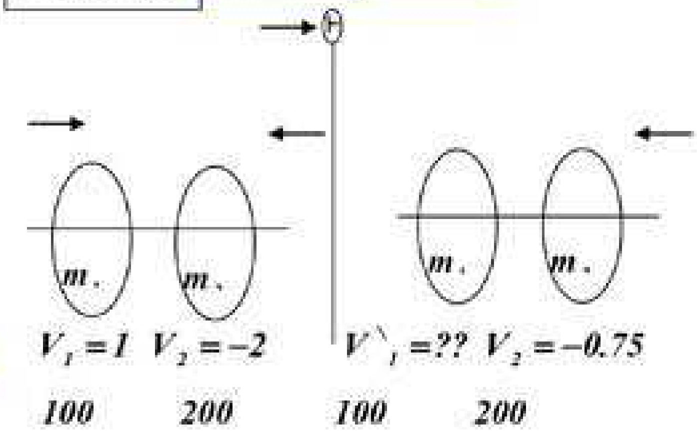
$0 = U^2 - 2 \times 4.9 \times 40$

$U = 14\sqrt{2}\ \ m\ /\ sec.$

Impulse & Collision

Example *Two smooth spheres of masses 100 gm, 200 gm are moving on horizontal ground in the same st. line. The velocity of the first is 1 m /sec. and the velocity of the second is 2 m / sec in an opposite direction, if the two spheres are collide, so that the second sphere moves in the same direction with velocity 0.75 m /sec, after impact, Find the velocity of the first sphere and the [...]e second sphere on it.*

Solution



Before *after*

$m_1V_1 + m_2V_2 = m_1V_1^{\backslash} - m_2V_2^{\backslash}$

$100(1) + 200(-2) = 100\ (V^{\backslash}_1) + 200\ (-0.75)$

$\therefore\ = -1.5\ m\ /\ sec$ *opposite in direction*

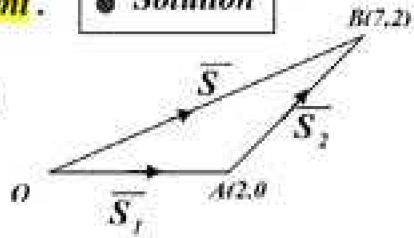
The impulse of the second sphere

$= m_1(V_1^{\backslash} - V_1)$

$= 100\ (-150 - 100) = -25000\ \ dyne\ .\ sec$

Example A particle moves from the origin $o = (0, 0)$ to the point $A(2, 0)$ Along a st. line, then to the point $B = (7, 2)$ on a st. line also under the action of a force $\overline{F} = -4\hat{i}$. Calculate the work done by this force during each of these two displacement , then prove that the sum of the two works is equal to the work done along the resultant displacement .

Solution



$\overline{S}_1 = \overline{OA} = (2, 0)$

$w_1 = \overline{F} \cdot \overline{S_1} = (-4, 0) \cdot (2, 0) = -8$ unit

$\overline{S}_2 = \overline{AB} = \overline{OB} - \overline{OA} = (7, 2) - (2, 0) = (5, 2) = (-4, 0) \cdot (5, 2) = -20$ unit

the sum of the two works $= w_1 + w_2 = -8 - 20 = -28$ unit

The resultant displacement $\overline{S} = \overline{OB} = (7, 2)$

$w = \overline{F} \cdot \overline{S} = (-4, 0) \cdot (7, 2) = -28$ unit

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Example A man ascends a st. road inclined at an angle of 30 to the horizontal, he moved a distance 300 m then he returned to the starting point. Find the work done by its weight through out the whole journey, if the resistance force to the motion of the man is equal to 2 kg.wt during his motion find the work done by this force during the whole journey.

Solution

/ The man returned to the starting point

$\therefore$ the total displacement $= \vec{0}$

$\therefore w = \overline{F} \cdot \overline{S} = mg \sin\theta \times 0 = \boxed{0}$

the resistance is always opposite to the motion.

/ the resistance opposite to the motion

$\therefore$ the work done by resistance

$= -RS$, $S = Vt$

$= -15 \times 150$ $= (9 \times \frac{5}{18}) \times 60$

$= -2250$ kg.wt.m $= 150$ m

Example A bullet of mass 25 gm is projected with horizontal velocity of magnitude 5.6 cm/sec^2 at a vertical target of wood. It embed through it and comes to rest after sec. Calculate the work done by the resistance of wood during this interval.

Solution

Inside the wood.

$V = U + at$

$0 = 5.6 + a(\frac{1}{35})$

$\therefore a = -196\ m/sec^2$

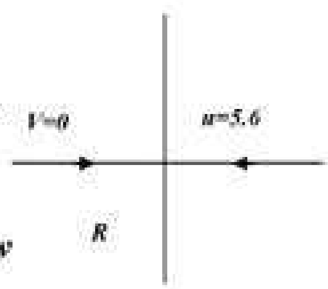
$V^2 = U^2 + 2as$

$0 = (5.6) - 2 \times 196\, s$

$\therefore s = 0.08$ m

From newton's 3^{rd} law

$\therefore -R = ma$



$$\ldots \quad -R = 0.025 \times -496$$

$$\ldots \quad R = 4.9 \text{ newton}$$

Work $w = -RS = -4.9 \;\; 0.08 = -0.392$ joule $= 392 \times 10^4$ reg

The power

A train of mass 150 tons, the power of its engine is 392 k . watt, moves along a horizontal straight road whose resistance to its motion is 0.02 from the weight of the train. Find the maximum velocity of the train. Solution

The power = 392 k . watt

$$= 392 \times 10^3 \text{ watt} = \frac{392 \times 10^3}{9.8} = 40000 \text{ kg . wt .}$$

m/sec

The resistance $= 0.02 \times 150 \times 1000$

$= 3000$ kg . wt

At the max. velocity , the motion is uniform

$F = R \Rightarrow \quad F = 3000$ kg . wt (driving force

the power $= F \times V \qquad 40000 = 3000 \times V$

$$V = \frac{40}{3} m / sec .$$

Example An airplane flies in a horizontal st. line under the action of a resistance proportional to the square of its velocity if the power of the maximum velocity of the airplane is 540 horse and the maximum velocity of the airplane is 360 km / h . Find the resistance when its velocity becomes 240 km / h.

Solution

$$V_{max} = 360 \times \frac{5}{18} = 100 \, m / sec .$$

$$V_2 = 240 \times \frac{5}{18} = \frac{200}{3} m / sec .$$

At the max. velocity $\quad F = R$

$\ldots$ the power $= R \times V_{max}$

$$540 \times 75 = R \times 100 \quad \Rightarrow \quad R_{max} = 405 \, kg.wt$$

$$\because \; R \;\; \alpha \;\; V_2$$

$$\therefore \quad \frac{R_{max}}{R_2} = \frac{V_{max}^2}{V_2^2}$$

$$\therefore \quad \frac{405}{R_2} = \frac{(100)^2}{(\frac{200}{3})^2}$$

$$\therefore \quad R_2 = \frac{405 \times 40000}{10000 \times 9} = 180 \, kg.wt$$

Example A train of mass 75 tons , and the power of its engine is 120 horse moves on a horizontal road with its max. velocity which is 72 km / h. Find the road resistance to its motion. Find also the max. velocity of the train when it ascends a road inclined to the horizontal at an angle of sine assuming that the resistance is the same on the two roads. Solution

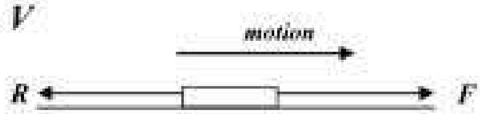
1^{st} : The motion on a horizontal road

Power $= 120 \times 75 = 9000$ kg . wt . m / sec.

$$V_{max} = 72 \times \frac{5}{18} = 20 \, m / sec$$

at the max. velocity $\quad F = R$

$\ldots$ power $= R \times V$



$$9000 = R \times 20 \Rightarrow R = 450 \ kg . wt$$

2^{nd} : The motion on the inclined road

At the max. velocity

$$F = R + mg \sin \theta$$

$$\therefore \quad = 450 \times 9.8 + 75 \times 1000 \times 9.8 \times \frac{1}{1000}$$

$$= 11760 \ newton \qquad = 1200 \ kg . wt$$

$\because$ The power doesn't change at the max. velocity

$$9000 = 1200 \times V \qquad V = 7.5 \ m/sec$$

Example A constant force $\overline{F} = 3\hat{i} + 4j$ measured in units of dune , acts on body. The displacement vector of the body is $\overline{S} = t\hat{i} + (\frac{1}{2}t^2 + t)j$ where t is measured by second and S by cm. Find the power of at the instant $t = 2$.

$$w = \overline{F} \ . \ \overline{S}$$

$$= (3\hat{i} + 4j) \ . \ (t\hat{i} + (\frac{1}{2}t^2 + t)j)$$

$$= 3t + 2t^2 + 4t$$

$$= 2t^2 + 7t$$

$$\text{The power} = \frac{dw}{dt}$$

$$= 4t + 7$$

at $t = 2$ The power $= 4(2) + 7 = 15$ erg / sec.

Example A const force acts on a body and the displacement of the body is given by $\overline{F} = (t^2 + 1)\hat{i} - 3t j$ if the power of the force $\overline{F}$ equals 1 **Solution** $t = 2$ and its power equals 24 erg / sec. at $t = 3$. Find

Solution

Let $\overline{F} = x\hat{i} + y j$

$$w = \overline{F} \ . \ \overline{S}$$

$$= (x(t^2 + 1) - 3yj$$

$$\text{The power} = \frac{dw}{dt}$$

$$= 2tx - 3y$$

at $t = 2 \Rightarrow$ power $= 14$

$$\therefore 4x - 3y = 14 \qquad (1)$$

at $t = 3 \Rightarrow$ power $= 24$

$$\therefore 6x - 3y = 24 \qquad (2)$$

Subtract (1) from (2)

$$\therefore 2x = 10 \qquad \boxed{x = 5} \quad , \quad \boxed{y = 2}$$

$$\therefore \overline{F} = 5\hat{i} + 2j$$

Example A car of mass 3 tons moves on a road inclined to the horizontal at on angle of $\sin \frac{1}{50}$ if the car ascends the road with max. velocity 22.5 km / h and it descends the dame road with max. velocity 90 km / h. determine the magnitude of the resistance of the road to its motion assuming that it is the same in each motion and the power of the engine of the car.

Solution

1st : when the car ascends

$$V_{max} = 22.5 \times \frac{5}{18} = 6.25\ m / sec$$

$$F = R + mg \sin\theta$$

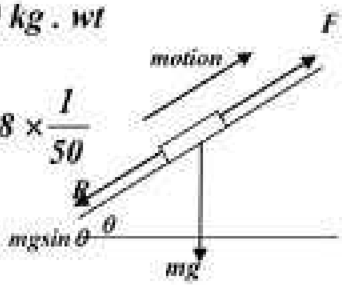
at the max. velocity $F = R$

$\therefore$ power $= R \times V$

$9000 = R \quad 20 \quad R = 450\ kg . wt$

$\therefore \quad F = R + 3 \times 1000 \times 9.8 \times \frac{1}{50}$

$\therefore \quad F = R + 588$ newton



$\therefore$ the power $= F \times V_{max}$

$= (R + 588) \times 6.25 = 6.25\,R + 3675 \quad (1)$

1st : when the car descends

$F^{\backprime} + mg \sin\theta = R$

$\therefore\ F^{\backprime} = R - 588$ newton

thepower $= F^{\backprime} \times V^{\backprime}_{max} = (R - 588) \times 25$

$= 25R - 14700 \qquad (2)$

$\because$ the power doesn't change

$\therefore\ 25\,R - 14700 = 6.25\,R + 3675$

$\therefore\ R = 980$ newton $= 100\ kg . wt$

Subst. in (1) $\therefore$ the power $= 6.25 \quad 980 + 3675$

$= 9800$ newton . m / sec $= 1000$ kg . wt .

m / sec $= 13\frac{1}{3}$ horse

Kinetic energy

Example A body of mass 14 kg. moves along a st. line and its displacement vector during the time interval (t) is given by $\overline{S} = (\frac{1}{3}t^3 - t^2 + 14)\hat{c}$ where $\hat{c}$ is a unit vector in the direction of its motion and t is measured in seconds and the magnitude of $\overline{S}$ is measured in meters. Find the increase of the kinetic energy of the body between $t = 4$ second and $t = 7$ second.

Solution

$$\overline{V} = \frac{d\overline{s}}{dt} = (t^2 - 2t)\hat{c}$$

at $t = 4$

$$V_1 = (4^2 - 2(4)) = 8 \quad m / sec$$

$$T_1 = \frac{1}{2} m V_1^{\,2}$$

$$= \frac{1}{2} \times 14 \times (8)^2 = 448\ joule$$

at $t = 7$

$$V_2 = (7^2 - 2(7)) = 35 \quad m / sec$$

$$T_2 = \frac{1}{2} m V_2^{\,2}$$

$$= \frac{1}{2} \times 14 \times (35)^2 = 8575 \quad joule$$

$\therefore$ the increase of kinetic energy =

$T_2 - T_1 = 8575\ -\ 448 = 8127$ joule